

# Model experiments illustrating fibre pull-out

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Pull-out of a rigid fibre from a solid matrix has been studied in a model of a composite material using transparent media to display the fracture. Solidified gelatine solution was used to model an elastic-plastic matrix and silicone rubber used to represent a brittle elastic material. The conditions dictating both brittle and plastic failure in the model have been presented theoretically and supported by experimental observations. In addition, a novel type of failure, where bubble cracks propagated between fibre and matrix, was observed.

## 1. Introduction

The failure of fibre reinforced materials by pull-out of the fibres from the matrix has been noted in numerous cases [1-4]. In consequence, the mechanics of fibre pull-out have generated much interest theoretically [5]. From the practical standpoint, too, fibre pull-out has been recognized as a limiting factor governing the strength of composites. The widespread use of pull-out tests in assessing the compatibility of fibre and matrix may be taken as evidence of this [6, 7].

Evidently, from published reports, the pull-out process is complex. For metal matrices, failure may usually be associated with plastic deformation [2] of the matrix whereas with polymeric composites, interfacial debonding of a brittle nature is often observed [8, 9]. The plastic pull-out problem has already been dealt with at some length [10]. By contrast, the condition dictating brittle pull-out has received relatively little attention.

The purpose of this report is to present a criterion for brittle failure during fibre pull-out and to contrast this with the plastic failure condition. Supporting experimental evidence derived from model testing is also given. In addition, a new type of debonding failure, in which bubble cracks were seen to move along the fibre-matrix interface, is described.

## 2. The pull-out model

Fig. 1a illustrates the pull-out model. Plexiglas

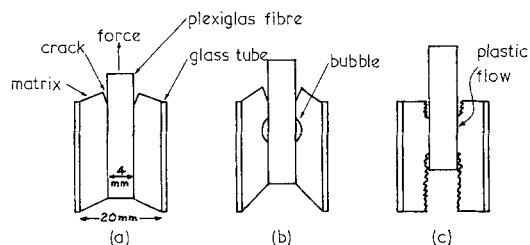


Figure 1 Observed modes of failure in the pull-out model.

was chosen as a relatively rigid material, silicone rubber as a representative elastic matrix and solidified aqueous gelatine solution as an elastic plastic substance. These materials have been used previously in studies of brittle interfacial failure [11, 12].

A glass tube was filled with the molten gelatine solution or uncured rubber and a Plexiglas rod was immersed along the axis of the tube until solidification of the matrix material occurred. In this model the axial rod simulates a rigid fibre embedded in a solid matrix and the glass tube represents the constraining effect of surrounding fibres in the composite.

Preliminary experiments showed how failure occurred in this model when the fibre was subjected to an axial pull-out force. First a crack was observed (Fig. 1a) travelling along the fibre. However, this crack would only propagate a short distance: at a certain point, the material behind the crack closed up around the fibre and left a bubble of detachment (Fig. 1b) which

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could itself propagate along the interface. For the purely elastic matrix this bubble was the cause of failure. The plastic material, although exhibiting these bubble cracks, failed by flow (Fig. 1c) as the pull force was further increased.

In the next sections, these three types of failure will be discussed in more detail.

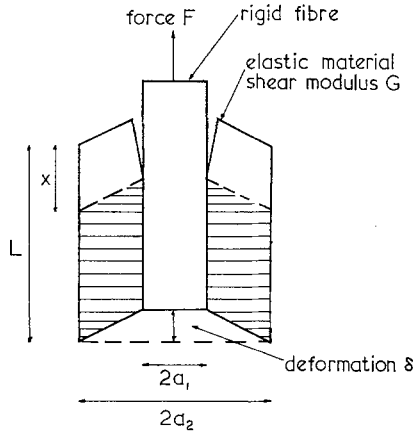


Figure 2 Theory of pull-out cracking.

### 3. Crack propagation theory

Imagine a crack of length  $x$  propagating along a fibre originally embedded to a depth  $L$  in the matrix (Fig. 2). We may derive a criterion for propagation of this crack using an energy balance method [13] where strain energy in the elastic material and potential energy in the applied load are converted exactly into surface energy as the crack proceeds. The problem involves the calculation of the sum of strain, potential and surface energy terms, then making the differential of this sum zero with respect to crack length.

Using the nomenclature of Fig. 2 the energy terms may be calculated. The surface energy term is given quite simply by  $2\pi a_1 xR$  where  $R$  is the adhesive energy per unit contact area. The elastic and potential energy terms may be calculated knowing that the movement  $\delta$  of the rigid fibre under the force  $F$  is given by [14]:

$$\delta = \frac{F}{2\pi G(L-x)} \ln \left( \frac{a_2}{a_1} \right). \quad (1)$$

This equation gives the deformation due to the shaded region alone in Fig. 2. Other deforma-

tions will also exist in the unshaded regions but it is assumed here that these will make only a minor contribution because they are either very small or are independent of crack length such that they disappear upon differentiation below. The elastic energy term is:

$$\frac{F^2}{4\pi G(L-x)} \ln \left( \frac{a_2}{a_1} \right)$$

and the potential energy in the applied load will be twice this and negative, i.e.

$$-\frac{F^2}{2\pi G(L-x)} \ln \left( \frac{a_2}{a_1} \right).$$

After calculating the total energy in the system and making its differential with respect to  $x$  zero, we find the force to cause crack propagation in the pull-out test:

$$\frac{F}{(L-x)} = \left[ \frac{8\pi^2 a_1 G R}{\ln(a_2/a_1)} \right]^{\frac{1}{2}}. \quad (2)$$

We may note that this condition, in addition to describing adhesive failure, could also be used to describe cohesive failure in the matrix if cohesive fracture energy were substituted for the adhesive energy  $R$ .

### 4. Results for crack propagation

Equation 2 was tested experimentally using the solidified gelatine solution model. Values of the shear modulus,  $G$ , and the adhesive energy,  $R$ , were first obtained in preliminary experiments.

The shear modulus was measured by applying small axial loads, insufficient to cause cracking, to the fibre and measuring the deflection,  $\delta$ .  $G$ , calculated from these tests using Equation 1, varied from sample to sample but was typically about  $10^3 \text{ N m}^{-2}$ .

The adhesive energy,  $R$ , was determined in a subsidiary experiment using the ninety degree peeling test. A sheet of gelatine was first prepared by pouring warm aqueous solution onto a Plexiglas plate and allowing the liquid to solidify. The Plexiglas plate had previously been cleaned in the same way as the rods; by rubbing lightly with detergent and rinsing for a long time in tap water. It was assumed that the surface properties of rods and plate would be similar. Peeling a strip of this gel from the Plexiglas gave the peel strength values plotted in Fig. 3 over a range of peeling speeds. Since peel strength is related to adhesive energy by the equation [15]

$$\frac{P}{b} = R, \quad (3)$$

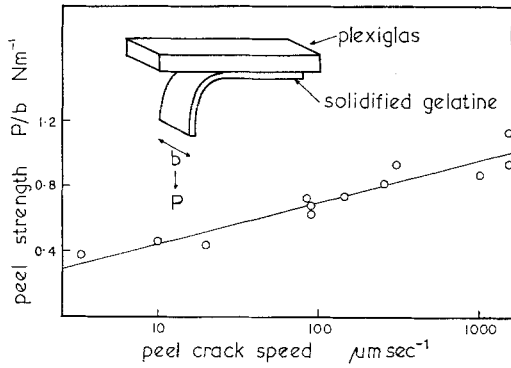


Figure 3 Measurement of adhesive energy, *R*.

the points shown in Fig. 3 represent values of adhesive energy for the gelatine Plexiglas system as a function of crack speed.

Cracking in the pull-out model was then investigated. Testing was carried out using an Instron machine, the axial fibre being pulled out a certain distance and then fixed in position. After manual initiation, a crack propagated along the rod at an approximately constant speed, which was measured visually.

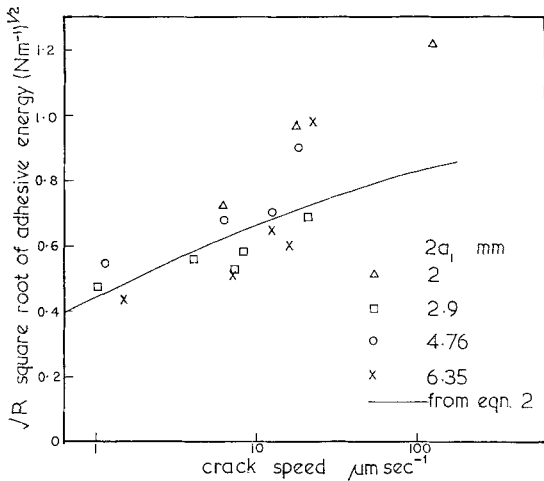


Figure 4 Results for pull-out cracking.

The results are plotted in Fig. 4 in terms of  $\sqrt{R}$  calculated from Equation 2 using the experimental measurements. Also shown is the theoretical line obtained from the data of Fig. 3. Reasonable agreement was obtained considering the scatter in the observations. Additional results

using silicone rubber as the matrix gave similar agreement.

**5. Bubble formation**

The pull-out crack did not travel very far along the fibre. After proceeding a distance of about  $3(a_2 - a_1)$  the crack appeared to slow down. Shortly after this, the material behind the crack tip closed up around the fibre to form a bubble which continued to move along the fibre at about the same speed as the crack had moved previously.

This observation of bubble formation was most interesting. On reflection, it seemed obvious that an ordinary crack could not continue to propagate along the fibre matrix interface since the separation between the fibre and the enclosing glass tube remained fixed. Clearly, the mode of cracking considered in the previous sections can only exist near the end where the fibre emerges from the matrix. This end crack must eventually stop but can then inject bubble cracks into the fibre matrix interface. These bubble cracks, rather reminiscent of the Schallamach waves [16] observed in friction studies, can then propagate along the interface to cause eventual failure.

The detailed mechanism of bubble crack propagation will be considered elsewhere [17].

**6. Plastic flow and friction**

In the gelatine model, the bubble cracks did not propagate for a very long distance along the fibre. Instead the bubbles gradually decreased in size until they disappeared altogether. A subsequent increase in force applied to the fibre then caused plastic yielding of the gel and the fibre could be withdrawn.

If the flow stress is  $\tau$ , then the condition for plastic pull-out is

$$\frac{F}{(L - x)} = 2\pi a_1 \tau \tag{4}$$

In this case the pull-out force should increase in proportion to the fibre radius. Fig. 5 shows some results illustrating this point.

After yield, the force required for pull-out dropped considerably. However, this frictional pull-out force was still significant and again followed Equation 4 but with a reduced flow stress (Fig. 5).

**7. Brittle or plastic?**

Having presented the criteria for both brittle and plastic pull-out failure, it is now possible to

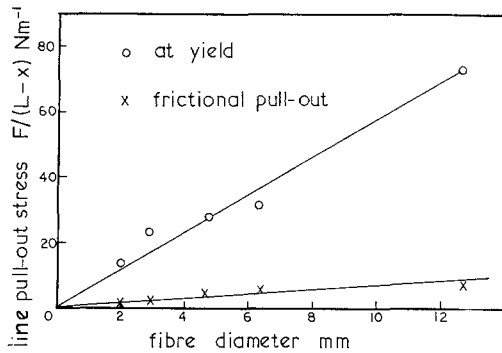


Figure 5 Pull-out force under plastic and frictional conditions.

decide when each type of failure should occur. Equation 2 must first be modified slightly to take into account the fact that

$$\frac{\pi a_1^2}{\pi a_2^2} \approx \frac{\text{volume of fibres}}{\text{volume of composite}} = V_f \quad (5)$$

where  $V_f$  is the volume fraction of fibres. Equation 2 for brittle pull-out may now be written

$$\frac{F}{(L-x)} = \left[ \frac{16\pi^2 a_1 G R}{-\ln V_f} \right]^{\frac{1}{2}} \quad (6)$$

At constant volume fraction of fibres, therefore, the brittle pull-out force is proportional to the square root of fibre radius whereas the plastic pull-out force depends directly on fibre radius. Consequently, for large diameter fibres a smaller force will be required for brittle fracture so that brittle failure should predominate. For small diameter fibres, on the other hand, ductile failure will be preferred. Combining Equations 4 and 6 it may easily be shown that the transition between brittle and ductile failure should occur at a fibre radius

$$a_1 = \frac{4GR}{-\tau^2 \ln V_f} \quad (8)$$

## 8. Conclusions

The different types of failure occurring when a rigid fibre is pulled from a solid matrix have been

studied using a transparent model. Three kinds of failure were observed:

(a) brittle fracture near the point where the fibre emerged from the matrix;

(b) bubble fracture where an enclosed region of detachment could be formed from the end crack and could be propagated along the fibre;

(c) plastic failure when the yield stress of the matrix material was exceeded.

Criteria for brittle and plastic failure have been presented and it has been shown that whereas at low fibre diameters plastic failure should predominate, at larger fibre diameters, brittle fracture should occur.

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